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Reaction pp → pp$\pi\pi\pi$ as a background for hadronic decays of the η' meson

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Abstract. Isospin violating hadronic decays of the η and η' mesons into 3π mesons are driven by a term in the QCD Lagrangian proportional to the mass difference of the d and u quarks. The source giving large yield of the mesons for such decay studies are pp interactions close to the respective kinematical thresholds. The most important physics background for $\eta, \eta' \to \pi\pi\pi$ is coming from direct three-pion production reactions. In case of the η meson the background for the decays is relatively low ($\approx 10\%$). The purpose of this article is to provide an estimate of the direct pion production background for the $\eta' \to 3\pi$ decays. Using the inclusive data from the COSY-11 experiment we have extracted the differential cross-section for the $pp \to pp$-multipion production reactions with the invariant mass of the pions equal to the η' meson mass and estimated an upper limit for the signal to background ratio for studies of the $\eta' \to \pi^+\pi^-\pi^0$ decay.

1 Motivation

1.1 Three-pion decays of the η and η' mesons

The η and η' decays into three pions violate isospin and occur only due to u and d quark mass difference. The decay width is sensitive to the mass difference: $\Gamma_{\eta(\eta')\to\pi^+\pi^-\pi^0} \propto \Gamma_0 \cdot (m_d - m_u)^2$, where the Γ_0 term can be calculated in the isospin limit $m_d = m_u$. The decays might provide a precise constraint for the light-quark mass ratios [1].

In the case of the η' meson the existence of the isospin conserving decays into three pseudoscalars ($\pi\pi\eta$) implies that instead of the decay width for the isospin violating $\pi\pi\pi$ channel one can measure the ratio

$$\frac{BR(\eta' \to \pi\pi\pi)}{BR(\eta' \to \pi\pi\eta)}, \qquad (1)$$

as it was proposed by Gross, Treiman and Wilczek [2]. Such measurement is self-contained since it does not require normalization for the partial decay width from other experiments. Additionally, simultaneous measurement of the two decays modes, with similar final states, ensures that many systematic uncertainties will cancel. The $\eta' \to \pi\pi\pi$ decays provides also a very sensitive test of the Chiral Perturbation Theory (CHPT) framework [3,4] extensions to the η' meson. Due to the large mass of the η' meson, the decays are strongly influenced by light-vector and scalar-meson resonances. Those decays cannot be studied using standard CHPT methods. An elegant method for accounting for the nonperturbative effects in two pseudoscalar meson interactions is given by unitarization procedure of the one loop CHPT result. For example in the theoretical studies of the η and η' meson decays the rescattering of any pair of the pseudoscalar mesons is described by Bethe-Salpether equations [5–7]. The parameters of the interactions are obtained by fits to the pseudoscalar scattering data. Predictions for many η and η' decays were given using the above technique within so-called *chiral unitary approach* [8].

The experimentally determined value of the branching ratio of the $\eta' \to 3\pi^0$ decay is $(1.56 \pm 0.26) \times 10^{-3}$ [9]. The decay into $\pi^+\pi^-\pi^0$ was observed in 2009 for the first time by the CLEO Collaboration and the branching ratio was determined to be $(37^{+11}_{-9} \pm 4) \times 10^{-4}$ [10]. The result is in strong disagreement with the value 10^{-2} predicted within the chiral unitary approach. Much more improvement on the theory and experiment side is needed to understand the three pion decays of the η' meson.

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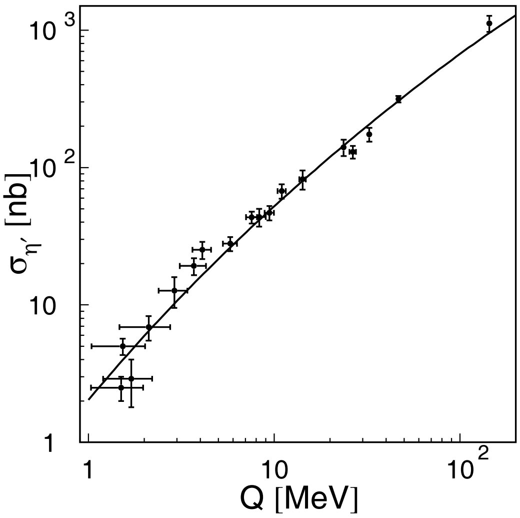

Fig. 1. Compilation of the data for the $pp \to pp\eta'$ reaction cross-section from COSY-11, DISTO and SPESIII measurements [22–24,26,27]. The solid line is parameterization of the data using formula (2).

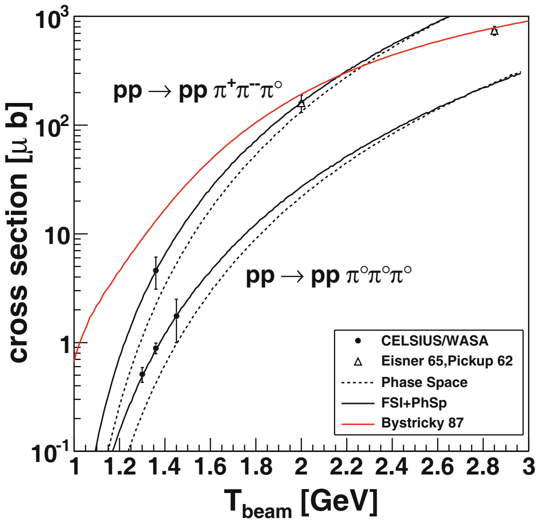

Fig. 2. Total cross-section for three-pion production: data and parameterizations. The data are from [43–46,32]. The parameterizations are from Bystricky *et al.* [47] and Fäldt and Wilkin [28,29]. The kinetic beam energy at thresholds for the η and η' production is equal to 1.255 GeV and 2.404 GeV, respectively.

1.2 Experiments using $pp \to pp\eta', \eta$ reaction

The mesons for the decay studies are produced in γp [11, 12], pp [13,14], $\pi^- p$ [15,16], pd [17–19] or e^+e^- [20,21] interactions. For the studies at light ion storage rings as COSY the $pp \to pp\eta'$ reaction close to threshold seems to be most promising. The cross-section for the $pp \to pp\eta'$ reaction was measured by the COSY-11 [22–25], SPESIII [26] and DISTO [27] Collaborations. In fig. 1 the experimental data are compared to the analytical parameterization derived by Fäldt and Wilkin [28,29], which takes into account final-state interaction of the protons

$$\sigma_{\eta'}^{tot}(Q) = C\frac{Q^2}{m_p p_{LAB}}\frac{1}{1+\sqrt{1+\frac{Q}{\epsilon}}}, \qquad (2)$$

where Q denotes the excess energy, p_{LAB} the beam momentum and m_p the proton mass. In comparison to the pp interaction the p-η' interaction is negligible [30]. The C and ϵ free parameters were determined by a fit to the experimental data [31]: $\epsilon = 0.62 \pm 0.13\,\text{MeV}$ and $C = 42 \pm 7\,\text{mb}$.

The experience from the studies of the $\eta \to \pi\pi\pi$ decays with η produced in $pp \to pp\eta$ reaction at beam energies 1.30–1.45 GeV carried out by the CELSIUS/WASA Collaboration shows that the background from direct three-pion production is about 15% for the $\pi^+\pi^-\pi^0$ channel and about 5% for the $3\pi^0$ channel [32,33]. This allows a precise study of the η decays providing a large number of events is collected. The production cross-section for the η mesons in pp colisions [34–37,13] is about 30 times larger than the cross-section for the η' meson [22–27] at the corresponding excess energies. At the same time the total cross-section for the direct three-pion production increases about two orders of magnitude between η and η' production thresholds (fig. 2). For the $pp \to pp\pi^+\pi^-\pi^0$ reaction total cross-section there are only three experimental points in the beam kinetic energy range up to 3 GeV. The cross-section for the $pp \to pp\pi^0\pi^0\pi^0$ reaction near the η' meson production threshold was not measured at all. However, based on the statistical model [38–40] and the isobar model [41, 42] one expects the $pp \to pp\pi^0\pi^0\pi^0$ cross-section to be 6–10 times lower than for the $pp \to pp\pi^+\pi^-\pi^0$ reaction. This is in agreement with an extrapolation of the CELSIUS/WASA measurement of both the reactions close to η meson production threshold. For the estimate of the background for the three-pion decays of the η' mesons instead of the total cross-section the relevant quantity is the value of the differential cross-section for the invariant masses of the pions in the range of the η' meson mass. This quantity was not measured and in the present paper we will provide an estimate for the upper limit of the background using the COSY-11 data where only outgoing protons were registered.

2 General considerations

Let us consider an example analysis chain leading to a selection of the $\eta' \to \pi^+\pi^-\pi^0$ decay from the $pp \to pp\eta'$ reaction. In the first step all tracks are reconstructed and particles identified. The events containing two protons from the production process, two charged pions and two photons are selected. Now one can apply energy-momentum conservation and select only events consistent with $pp \to pp\eta' \to pp\pi^+\pi^-\pi^0 \to pp\pi^+\pi^-\gamma\gamma$ reaction hypothesis. This procedure can be most generally implemented by kinematic fitting, but one can use also some other method. In the end the selection could be represented by a region in some control variable μ. For example μ could be missing mass squared or χ^2 value of the kinematic fit. Within the selected region, in addition to the signal, a contamination from the background events originating from reactions which have similar final states is unavoidable. The most important physics background channels for the discussed case is the direct $pp \to pp\pi^+\pi^-\pi^0$ reaction and other η' decays like $\eta' \to \pi^+\pi^-\eta \to \pi^+\pi^-\gamma\gamma$. Candidates for the identification variable (in addition to χ^2 of the kinematic fit) are in these cases, respectively: the missing mass of the two protons and the invariant mass of the two photons. In this article we focus on the first case: estimate the signal-to-background ratio and its implications for the statistical uncertainty of the extraction of $BR(\eta' \to \pi^+\pi^-\pi^0)$ value.

The signal-to-background ratio, N_S/N_B, can be written as

$$\frac{N_S}{N_B} = \frac{\sigma_{\eta'} \cdot BR \cdot \varepsilon_S \cdot \mathcal{L}}{\Delta\mu \cdot \rho_B \cdot \varepsilon_B \cdot \mathcal{L}}, \tag{3}$$

where the factors are:

1) $\sigma_{\eta'}$ the total cross-section for the production reaction (here for $pp \to pp\eta'$),
2) ρ_B the differential cross-section for the direct $\pi^+\pi^-\pi^0$ production with the pions invariant mass equal to the mass of the η' meson

$$\rho_B \equiv \left.\frac{\mathrm{d}\sigma_B}{\mathrm{d}\mu}\right|_{\mu=m_{\eta'}}, \tag{4}$$

3) ε_S, ε_B acceptances and reconstruction efficiencies for the signal and the background,
4) $\Delta\mu$ a range of the missing mass used for the extraction of the signal which depends on the detector resolution,
5) BR the measured branching ratio of the $\eta' \to \pi^+\pi^-\pi^0$ decay,
6) $\mathcal{L}$ the integrated luminosity.

The N_S/N_B ratio depends on the beam energy through $\sigma_{\eta'}$, ρ_B, the missing-mass resolution and the detection efficiencies. Hereafter we derive the energy dependence of these quantities.

3 Background estimate

The value of the ρ_B cross-section should be determined from the $\pi^+\pi^-\pi^0$ invariant-mass distributions of the direct pion production reaction. However, there is no data at beam energies near the η' meson threshold. Therefore, we estimate an upper limit for the ρ_B by re-evaluating the available missing-mass spectra of the $pp \to ppX$ reaction determined by the COSY-11 Collaboration at several beam energies near the the η' threshold [22–25]. In fig. 3 an example of the COSY-11 reconstructed missing-mass distribution at the $pp \to pp\eta'$ excess energy, Q, of 15.5 MeV is shown [31].

Let us consider a measurement of the two protons from the $pp \to ppX$ reaction, where there are no constrains on other outgoing particles. In a first approximation the ratio of the number of the background events in a slice under the η' peak in the missing-mass spectrum to the number of events in the peak does not depend on the detector acceptance for the protons. This assumption is valid, for example, if the η' meson and the multipion reactions are simulated according to phase-space or even if an universal final-state interaction between protons is introduced [48,49]. The assumption may break, for example, if there is a significant difference in four momentum transfer distributions between η' and multimeson production. However, the dependence of the production amplitude on the momentum transfer is very weak near the kinematical threshold.

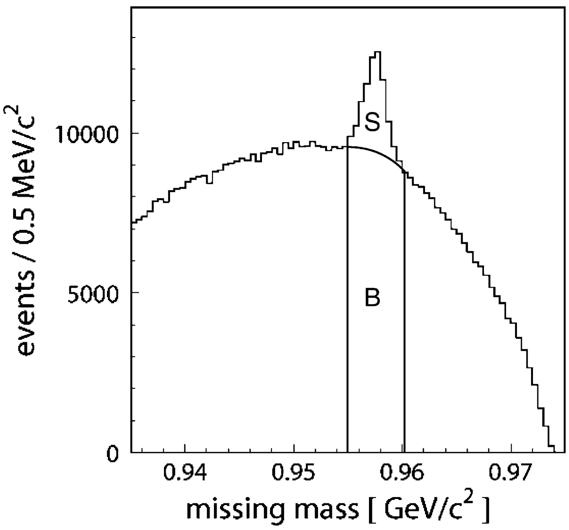

Fig. 3. An example of the missing-mass distribution for the $pp \to ppX$ reaction from the COSY-11 measurements at $Q = 15.5$ MeV [31]. S denotes the signal originating from the η' meson production and B indicates the background under the peak. The continuum originates from direct production of two, three and more mesons. It provides a conservative upper limit for the background from the $pp \to pp\pi^+\pi^-\pi^0$ reaction.

The differential cross-section ρ_B for the background originating from all multimeson channels was determined from the COSY-11 data according to the formula

$$\rho_B(Q) = \frac{N_B(Q)}{N_S(Q)} \frac{\sigma_{\eta'}(Q)}{\Delta\mu}, \tag{5}$$

which is derived from the following expressions for the $N_S(Q)$ and $N_B(Q)$:

$$N_S(Q) = \sigma_{\eta'}(Q) \cdot \varepsilon(m_{\eta'}, Q) \cdot \mathcal{L}, \tag{6}$$
$$N_B(Q) = \rho_B(Q) \cdot \Delta\mu \cdot \varepsilon(m_{\eta'}, Q) \cdot \mathcal{L}, \tag{7}$$

where N_S stands for the number of the observed events in the η' peak, N_B the number of the background events in the $\Delta\mu$ slice under the η' signal, $\sigma_{\eta'}$ denotes the total cross-section for the $pp \to pp\eta'$ reaction described according to analitical formula from eq. (2), ϵ denotes the combined acceptance and detection efficiency of the COSY-11 detector, which, in a very good approximation, depends

Table 1. Differential cross-section ρ_B for multimeson production in proton-proton collisions as a function of the $pp \to pp\eta'$ reaction excess energy Q. The ρ_B values were extracted from the experimental data [22–25] using eq. (5).

Q (MeV)	ρ_B (nb/MeV)	$\Delta\rho_B$(stat.) (nb/MeV)	$\Delta\rho_B$(syst.) (nb/MeV)
1.53	1.04	0.14	0.16
4.10	7.0	1.1	1.1
5.80	13.4	1.2	2.0
7.60	18.2	1.6	2.8
9.42	32.3	3.6	4.9
10.98	32.7	3.2	4.9
14.21	60	11	9
15.50	85	2.4	13
23.64	117	17	17
46.60	322	16	48

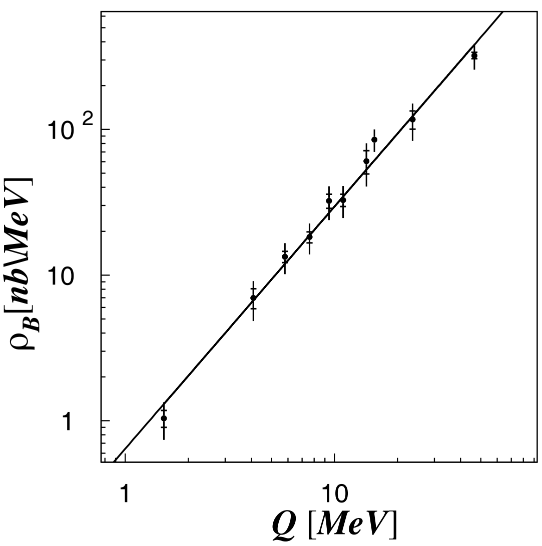

Fig. 4. Inclusive differential cross-section for multimeson production derived from the COSY-11 data [22–25]. Statistical and systematic errors are seperated by the horizontal bars. The superimposed line shows the function of eq. (8) fitted to the COSY-11 data.

only on the mass of the produced system and on the excess energy [48,49], $\mathcal{L}$ indicates the integrated luminosity. The width of the $\Delta\mu$ slice was selected to contain nearly 100% events of the signal peak. The derived values of ρ_B as a function of Q with the corresponding statistical and systematic errors are given in table 1 and are shown in fig. 4. The systematic uncertainties are discussed in [22–25].

The ρ_B dependence on the excitation energy is well described by the following parameterization:

$$\rho_B = \alpha(Q/Q_0)^\beta, \tag{8}$$

where $Q_0 = 1\,\text{MeV}$ is the normalization factor and α and β are the parameters. The fit gives $\alpha = 0.64 \pm 0.14\,\text{nb/MeV}$ and $\beta = 1.662 \pm 0.081$.

4 Statistical uncertainty of the BR measurement

Assuming that the shape of the background is known and is not correlated with the signal, the relative statistical uncertainty of the branching ratio can be expressed as

$$\frac{\sigma(BR)}{BR} = \frac{\sigma(N_S)}{N_S} = \frac{\sqrt{N_S + N_B}}{N_S}. \tag{9}$$

In our case $N_S \ll N_B$ and we have

$$\frac{\sigma(BR)}{BR} = \frac{\sqrt{N_B}}{N_S}. \tag{10}$$

Taking into account eq. (3) and eq. (10) one gets the formula for the statistical uncertainty

$$\sigma(BR) \le \frac{\sqrt{\rho_B \cdot \Delta\mu \cdot \varepsilon_B}}{\sigma_{\eta'} \cdot \varepsilon_S} \frac{1}{\sqrt{\mathcal{L}}}. \tag{11}$$

From the derived expression one sees that improved tagging resolution ($\Delta\mu$ decreased) helps only if the detection efficiency has not worsened.

The integrated luminosity $\mathcal{L}$ can be determined from the simultaneously measured decay $\eta' \to \pi^+\pi^-\eta \to \pi^+\pi^-\gamma\gamma$ with the well-established branching ratio. Therefore the statistical uncertainty of the luminosity determination can be neglected and many contributions to the systematic uncertainty in the ratio of signal to the monitoring events will cancel.

5 Feasibility for a large-acceptance detector

As an example application of the extracted ρ_B value and the formulas derived in the previous sections, we will consider a determination of the branching ratio for the $\eta' \to \pi^+\pi^-\pi^0$ decay using a large-acceptance detector. The large acceptance is necessary for an efficient identification of all outgoing particles. After the selection of the $pp\pi^+\pi^-\pi^0$ final state the direct three-pion production and the $\eta' \to \pi^+\pi^-\pi^0$ decay can be distinguished with the best precision using the missing mass of the two forward emitted protons.

For the calculations of the missing-mass resolution a typical beam and target parameters available at the Cooler Synchrotron COSY [50] were assumed: beam momentum spread $\Delta p/p \approx 10^{-3}$ (FWHM), perpendicular beam profiles (horizontal $\sigma_X = 2\,\text{mm}$, vertical $\sigma_Y = 5\,\text{mm}$) [51]. As a target, a hydrogen stream in a cylinder with diameter of 2.5 mm was used. The effective-energy resolution of the forward-scattered protons from the reaction $pp \to pp\eta'$ registered using plastic scintillators is typically in the order of few percent (3%).

The determined Q dependence of $\sigma(BR)/BR$ is shown in fig. 5, assuming one week experiment with luminosity of $L = 10^{32}\,\text{cm}^{-2}\,\text{s}^{-1}$ and value of $BR(\eta' \to \pi^+\pi^-\pi^0) = 0.37\%$ [10]. The optimum is reached for the excess energies

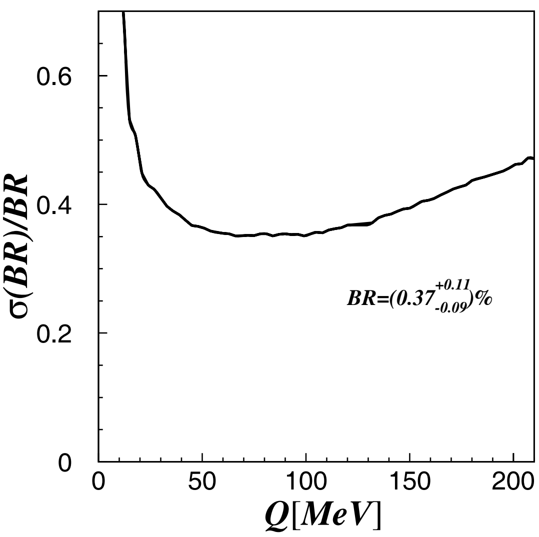

Fig. 5. The relative accuracy of the upper limit of the $BR(\eta' \to \pi^+\pi^-\pi^0)$ as a function of the excess energy Q for the $pp \to pp\eta'$ reaction.

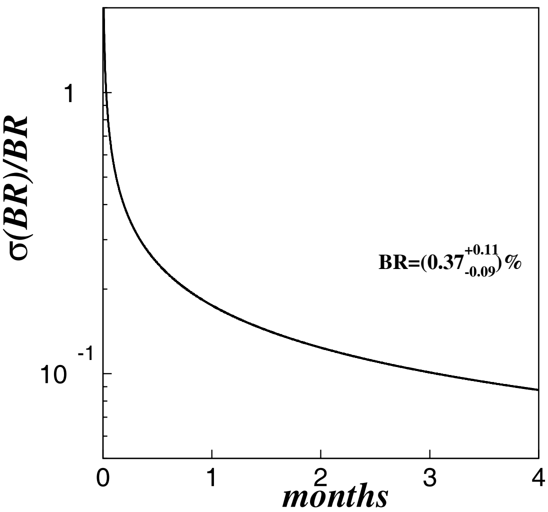

Fig. 6. The relative accuracy of the upper limit of the branching ratio for the decay $\eta' \to \pi^+\pi^-\pi^0$ as a function of the measurement time in months.

between 50 and 100 MeV. This is a general conclusion for a large-acceptance detection system. The statistical uncertainty of the branching ratio improves with the time of the measurement as $1/\sqrt{t}$. The dependence for the beam momentum of $p_{beam} = 3.45\,\mathrm{GeV}/c$ corresponding to the excess energy $Q = 75\,\mathrm{MeV}$ is shown in fig. 6. The plot indicates that for the BR equal 0.37% to achieve the relative accuracy of 10% would require at least a two-month experiment. However, signal-to-background ratio at this energy will be only about 10^{-3} which puts extreme requirements for the understanding of the systematic effects. Therefore the other strategy for the experiment would be to find a compromise between the statistical and systematic uncertanities by going to lower excitation energies where the signal-to-background ratio increases (fig. 7).

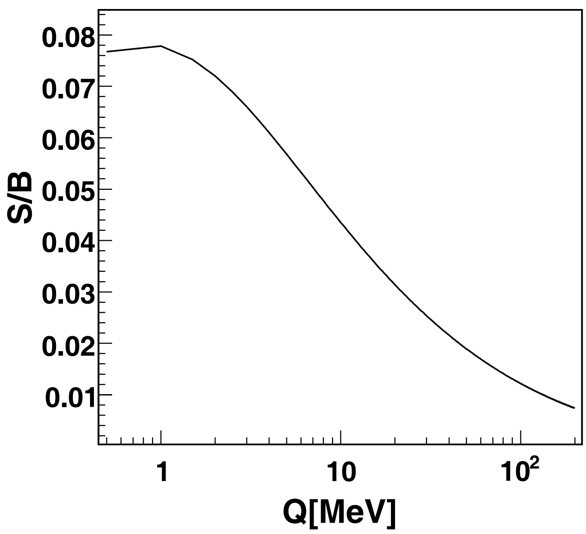

Fig. 7. The signal-to-background ratio $S : B$ calculated taking into account the natural width of the η' meson.

An additional source of background, not discussed here, comes from other decays of η' involving similar particles: $\eta' \to \pi^+\pi^-\eta$ and $\eta' \to \omega\gamma$. This background cannot be suppressed using the missing-mass method and the invariant masses of the decay products should be used instead.

6 Summary

Using the COSY-11 data for the $pp \to ppX$ reaction near the η' meson production threshold we extracted an upper limit for the background from the $pp \to pp\pi^+\pi^-\pi^0$ reaction for $\eta' \to \pi^+\pi^-\pi^0$ decay studies. This and the parameterization of the total cross-section energy dependence for the $pp \to pp\eta'$ reaction permit us to estimate that two months of the beam time with average luminosity of $10^{32}\,\mathrm{cm}^{-2}\,\mathrm{s}^{-1}$ would be sufficient to reach statistical accuracy of the previous experiments for the studies of $\eta' \to \pi^+\pi^-\pi^0$ using a large-acceptance detector. However, since the signal-to-background ratio for the optimal energy is only about 10^{-3}, one may expect a much larger systematic uncertainty. Since even taking the natural width of the η' meson the $S : B$ ratio is about 10^{-2}, and this points to inherent limitations of the $pp \to pp\eta'$ reaction for the $\eta' \to \pi^+\pi^-\pi^0$ decay shown in fig. 7. The situation is expected to be at least one order of magnitude better for the $\eta' \to \pi^0\pi^0\pi^0$ case. Also "not rare" decays $\eta' \to \pi\pi\eta$ can be studied in the $pp \to pp\eta'$ reaction.

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